

SECTION V

Assimilation of the Root's Coefficient (QUADRATIC EQUATION)

62-3) Assimilation of the root's coefficient ...

The sum or difference of a quantity and of a multiple of its square-root being given, the square of half the coefficient¹ is added to the given number; and the square-root of their sum [is extracted: that root,] with half the coefficient added² or subtracted being squared, is the quantity sought by the interrogator.³*

$$x \pm b\sqrt{x} = c, \quad \text{Find } x.$$

[Author's Note: You can easily identify this as: $x^2 \pm bx - c = 0$, replacing x by x^2]

Statement: coefficient, b ; given number, c

METHOD

- Sq. rt. {(half the coefft.)² + 'given number'}
- Square its **diff.** with half the coefft for $x + b\sqrt{x} = c$
or Square its **sum** with half the coefft for $x - b\sqrt{x} = c$

¹ mula, root; guna, multiplier; mula-guna, root's multiplier; coefficient of the root

² Add half the multiplier, if the difference were given; or subtract it, if the sum were so.

³ The quantity sought consists of two portions:
(1) sq rt taken into its multiplicator & (2) the given number

* This exact method, 'Add, square of, half of, coefficient of x ' to both sides ... is taught even today without any change and is called 'completing the square'



Ancient Indian Maths

64) Example (Difference is given, i.e. root subtracted) - So 'add' in step 4.

One pair out of a flock of gheese remained sporting in the water, and saw seven times the half of the sq.-rt. of the flock proceeding to the shore tired of the diversion. Tell me, dear girl, the number of the flock.

Flock less '7 times half' its root, 2. type: $x - b\sqrt{x} = c$

$$x - \frac{7}{2}\sqrt{x} = 2$$

Statement: multiplier, $\frac{7}{2}$; given number, 2

- | | | | |
|------------------------|----------------------|-----------------|---|
| 1) square | 'half of multiplier' | $\frac{49}{16}$ | |
| 2) add to | 'given' | $\frac{81}{16}$ | |
| 3) take sq root | | $\frac{9}{4}$ | |
| 4) ADD | 'half of multiplier' | 4 | $\left(\frac{9}{4}, \frac{7}{4}\right)$ |
| 5) square | the result | 16 | |

Gheese in the flock **16**.

Ganitham - (*Bhascara's Lilavaty*)

65) Example (Sum is given, i.e. root added) - So 'subtract' in step 4.

Tell me what is the number, which, added to nine times its sq.-rt., amounts to twelve hundred and forty?

$$x + b\sqrt{x} = c$$

$$x + 9\sqrt{x} = 1240$$

multiplier, 9; *given*, 1240.

- | | | | |
|---|-------------------------------|------------------|---|
| 1) square | 'half the <i>multiplier</i> ' | $\frac{81}{4}$ | |
| 2) add | ' <i>given</i> ' | $\frac{5041}{4}$ | (add 1240 x 4, 4800+160, 4960 to nr.) |
| 3) take sq root | | $\frac{71}{2}$ | |
| [Estimate sq rt of 5041 instantly; Refer <i>Quick Maths</i> by Prof Doss] | | | |
| 4) <u>SUBTRACT</u> | 'half the <i>multiplier</i> ' | 31 | $\left(\frac{71}{2} - \frac{9}{2}\right)$ |
| 5) square | the result | 961 | |

Required number is **961**.

Note:

Replace x by x^2 ; *bring c to the left*; we get the familiar quadratic equation

$$x^2 \pm bx - c = 0$$

In this case we get the value of x in the 4th step itself.
You may try some familiar quadratics.



Ancient Indian Maths

67) Example

The son of Pritha, irritated in fight, shot a quiver of arrows to slay CARNA. With half his arrows, he parried those of his antagonist; with four times the square-root of the quiver-full, he killed his horses; with six arrows, he slew SALYA; with three he demolished the umbrella, standard and bow; and with one, he cut off the head of the foe. How many were the arrows, which ARJUNA let fly?

Statement: *fraction* $\frac{1}{2}$, *multiplier* 4, *given* 10, *total* ? arrows

Modify by unity lessened by fraction 1, $\frac{1}{2}$; by $\frac{1}{2}$

New Statement: *multiplier* 8, *given* 20

- | | | |
|------------------------|-------------------------------|-----|
| 1) square | ‘half the <i>multiplier</i> ’ | 16 |
| 2) add | ‘ <i>given</i> ’ | 36 |
| 3) take sq root | | 6 |
| 4) add | ‘half the <i>multiplier</i> ’ | 10 |
| 5) square | the result | 100 |

Number of arrows **100**.