SECTION VI

PERMUTATION AND COMBINATION

110-112) [1]

Let the figures from one upwards, differing by one, put in the inverse order, be divided by the same [arithmeticals] in direct order; and let the subsequent be multiplied by the preceding, and the next following by the foregoing [result]. The several results are the changes, ones, twos, threes, &c. This is termed a general rule.[2]

It serves in prosody, for those versed therein, to find the variations of metre; in the arts [as in architecture] to compute the changes upon apertures [of a building]; and [in music] the scheme of musical permutations[3]*; in medicine, the combinations of different savours. For fear of prolixity, this is not [fully] set forth.*

[1] To find the possible permutations of long and short syllables in prosody; combinations of ingredients in pharmacy; variations of notes, etc. in music; as well as changes in other instances - GANESA

[2] Commentators appear to interpret this as a name of the rule here taught; *sadharana, or sadharana-chhando-ganita*, general rule of prosodian permutation: subject to modification in particular instances; as in music, where a special method (*asadharana*) must be applied. - GANESA, SURYADASA

[3] *Chanda-meru*: a certain scheme. - GANESA
It is more fully explained by other commentators: but the translator [Henry Thomas Colebrooke] is not sufficiently conversant with the theory of music to understand the term distinctly.

113) A single example in prosody:

In the permutations of the ga'yatri metre,[1] say quickly, friend, how many are the possible changes of the verse? and tell severally, how many are the combinations with one, [two, three,] etc. long syllables?

Here the verse of the *ga'yatri* stanza comprises six syllables. Wherefore, the figures from one to six are set down, and the statement of them, in direct and inverse order is $\begin{smallmatrix}6&5&4&3&2&1\\1&2&3&4&5&6\end{smallmatrix}$. Proceeding as directed, the results are, changes with one long syllable 6; with two 15; with three, 20; with four, 15; with five, 6; with six, 1; with all short, 1. The sum of these is the whole number of permutations of the verse, 64.[2]

In like manner, setting down the numbers of the whole tetrastic, in the mode directed,and finding the changes with one, two, &c. and summing them, the permutations of the entire stanza are found: 16777216[3]

In the same way may be found the permutations of all varieties of metre, from *Ucta'* [which consists of monosyllabic verses] to *Utcriti* [the verses of which contain twenty-six syllables.]

[1] The *Gayatri* metre in sacred prosody is a triplet comprising twenty-four syllables: as in the famous prayer containing the Brahmenical creed, called *gayatri*, (See As. Res. vol. 10, p. 463). But, in the prosody of profane poetry, the same number of syllables is distributed in a tetrastic: and the verse consequently contains six syllables. (As. Res. vol. 10, p. 469.)

[2] Changes with:

one long syllable: $\frac{6}{1}$, 6; two: $\frac{6 \times 5}{1 \times 2}$, 15; three: $\frac{6,5,4}{1,2,3}$, 20;

four: $\frac{6,5,4,3}{1,2,3,4}$, 15; four: $\frac{6,5,4,3,2}{1,2,3,4,5}$, 6; four: $\frac{6,5,4,3,2,1}{1,2,3,4,5,6}$, 1

[3] Each line is 64; four lines 64 x 64 x 64 x 64 = 16777216 - Author

Just for curiosity: $64 = 2^6$

So $16777216 = 2^{24}$

Again we know, $2^{10} = 1024$ (which is '1k' in computers.)

114) Example ...

In a pleasant, spacious and elegant edifice, with eight doors,[1] constructed by a skilful architect, as a palace for the lord of the land, tell me the permutations of apertures [window / doors] taken one, two, three, &c.[2] ...

Statement:
8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8

The number of ways in which the doors may be opened by ones, twos, or threes, &c. is 8, 28, 56, 70, 56, 28, 8, 1 and the changes on the apertures of the octagon[3] palace amount to 255.

114) ... Example

... Say, mathematician, how many are the combinations in one composition, with ingredients of six different tasted, sweet, pungent, astringent, sour, salt and bitter, taking them by ones, twos, or threes, &c.

Statement:
6 5 4 3 2 1
1 2 3 4 5 6

Number of various preparations with ingredients of diverse tasted is

6	15	20	15	6	1^4
1	2	3	4	5	6

[1] *Mucha*,aperture for the admission of air: a door or window; (same with *gacacsha;* - GANESA) a portico or terrace, (*bhumi-visesha*; - GANGADHARA, and SURYADASA)

[2] The variations of one window or portico open (or terrace unroofed) and the rest closed; two open, and the rest shut; and so forth.

[3] An octagon building, with eight doors (ow windows; porticos or terraces;) facing the eight cardinal points of the horizon, is meant. See Ganesa.

[4] Total number of possible combinations 63 - GANGADHARA

CHAPTER V

SREDHI - VYAVAHARA

PROGRESSIONS[1]

SECTION I

ARITHMETICAL PROGRESSION

115) Sums of the Arithmeticals ...

Half the period,[2] multiplied by the period added to unity, is the sum of the arithmeticals one, [two, three] *&c. and is named their addition.*

[1] *Sredhi*, a term employed by the older authors for any set of distinct substances or other things put together. - GANESA

It signifies sequence or progression. *Sredhi-vyavahara*,, ascertainment or determination of progressions.

[2] *Pada* the place. -GANESA The last of the numbers to be summed. - MANO

115) ... Aggregate of Sums

This being multiplied by the period added to two, and being divided by three is the aggregate of the additions.[1]

116) Example

Tell me, quickly, mathematician, the sums of the several [progressions of] numbers one, &c. continued to nine; and the summed sums of those numbers.

Statement:	Arithmeticals:	1	2	3	4	5	6	7	8	9
Answer:	Sums:	1	3	6	10	15	21	28	36	45
	Summed sums:	1	4	10	20	35	56	84	120	165

Demonstration

1) Statement: 1, 2, 3, ..., 9. Sum ?

Multiply '*half the period*', '*period increased by 1*'; $\frac{9}{2}$x10; **45.**

Aggregate of additions, Summed sums or Sum of sums;
is total of 1+ (1+ 2) + (1 + 2 + 3), etc.

2) Statement: 1, 2, 3, ..., 9. Sum of sums?

Sum of 1, 3, 6, 10, ..., 45;

Multiply the '*sum*' by '*period, increased by 2*' 45, 11
Divide by 3; 15, 11; **165.**

[1] The first figure is unity. The sum of that and the period being halved, is the middle figure. As the figures decrease behind it (middle figure), so they increase before it (middle figure): wherefore the middle figure, multiplied by the period, is the sum of the figures one, &c. continued to the period. - GANESA

It is a maxim, that '**a number multiplied by the next following arithmetical, and halved, gives the sum of the preceding:'** wherefore &c. - SURYADASA

CAMALACARA according to RANGANATHA:

1	2	3	4	5	6	7	
	7	6	5	4	3	2	1

Sum of each vertical pair is 8: wherefore this sum, multiplied by the number of terms, is twice the sum of the progression.

117) Sum of squares ...

Twice the period added to one and divided by three, being multiplied by the sum, is the sum of the squares. ...

Increase '*double* the *period'* by *one*
Multiply by the *sum*
Divide by *three*

$2n+1$
x S
$\div 3$

118) Example ...

Tell promptly the sum of the squares, and the sum of the cubes, of those number, if thy mind be conversant with the way of summation.

Statement: 1, 2, 3, ..., 9. Sum of squares?
i.e. $1^2 + 2^2 + 3^2 + ... + 9^2 = ?$

Increase '*double* the *period'* by *one*; 9, 1; 19 $2n+1$
Multiply by the *sum*: 19, 45 x S
Divide by *three*: 19, 45; 3 19, 15 **285** $\div 3$

117) ... Sum of cubes

... The sum of the cubes of the numbers one, etc. is pronounced by the ancients equal to the square of the addition.

Square the *sum* (Sum of cubes is Square of sum!)

118) ... Example

Statement: 1, 2, 3, ..., 9. Sum of cubes?
i.e. $1^3 + 2^3 + 3^3 + ... + 9^3 = ?$

Square the sum: 45^2; **2025**

119) Last Term, Mean and Sum

The increase[1] *multiplied by the period less one, and added to the first quantity, is the amount of the last. ...*

... That added to the first and halved, is the amount of the mean: which, multiplied by the period, is the amount of the whole, and is denominated (ganitha) the computed sum.

Demonstration

Statement: 5 9 13 17 21 25 29

Increase 4, *Period 7,* *first quantity* (first term) 5, *last* term 29

mean (middler term) 17, *sum 5 + 9 + 13 + ... + 29*

1) last term ?	**Multiply** *increase, 'period, less 1'*	$d(n-1)$
	Add *first term*	$+a$
2) mean ?	**Add** *first term, last term*	$a+l$
	Halve the *sum*	halve it
3) sum ?	**Multiply** *mean, period*	mn

[1] Where the increase is arbitrary - GANGADHEESWARA
In such cases, to find the last term, mean amount, and sum of the progression. - SURYADASA
From first term, common difference and period, to find the whole amount, &c. - GANESA

[2] *Adi*, and *mucha, vadana, vactra,* and other synonyma of face; the initial quantity of the progression; (that, from which as an origin the sequence commences. - SURYADASA) the first term.

Chaya, prachaya or uttara; the more (*adhica* - SURYA.) or argument (*vriddhi* - GANGADHARA) by which each term increases: the common increase or difference of the terms.

Antya; the last term.

Madhya; the middle term, or the mean of the progression.

Pada or *gachha*; the period, the number of terms.

Sarva-dhana, Sredhi-phala or *Ganita*; the amount of the whole; the sum of the progression.
'It is called *ganita*, because it is found by computation (*ganana*).' - GANESA

120) Example

A person, having given four drammas to priests on the first day, proceeded, my friend, to distribute daily alms at a rate increasing by five a day. Say quickly how many were given by him in half a month?

Statement: first 4; increase 5; period 15

1) last term ?	**Multiply** *5, 15-1; 70*	
	Add *4;*	**74**
2) mean ?	**Add** *4, 74; 78*	
	Halve the *sum,*	**39**
3) sum ?	**Multiply** *39, 15;*	**585**

121) Example[1]

The initial term being seven, the increase five, and the period eight, tell me, what are the numbers of the middle and last amounts? And what is the total sum?

Statement: first 7, increase 5, period 8

1) last term ?	**Multiply** *5, 7; 35*	
	Add *7;*	**42**
2) mean ?	**Add** *7, 42; 49*	
	Halve the *sum,*	**$24\frac{1}{2}$**
3) sum ?	**Multiply** *$24\frac{1}{2}$, 8;*	**196**

[1] To exhibit an instance of an even number of terms; where there can consequently be no middle term [but a mean amount]. - GANESA

122) First term ...

The sum of the progression being divided by the period, and half the common difference multiplied by one less than the number of terms, being subtracted, the remainder is the intial quantity. ...

first term ?

Divide *sum by period* $\frac{S}{n}$

Subtract *product of 'half increase', 'period, less 1'* $-\left\{\left(\frac{1}{2}d\right) \text{ x } (n-1)\right\}$

123) Example

We know the sum of the progression, one hundred and five; the number of terms, seven; the increase, three; tell us, dear boy, the initial quantity.

Statement: first ?, Com. diff. (*increae*) 3, Period 7, Sum 105.

Divide *105, 7;* 15

Subtract *product of 'half 3, 7-1*; 9; **6**

Initial quantity, **6**.

122) ... Increase

The sum being divided by the period, and the first term subtracted from the quotient, the remainder, divided by half of one less than the number of terms, will be the common difference.

common difference ? **Divide** *sum by period*

Subtract *first term*

Divide *by half* of *'period, less 1'*

$$\frac{S}{n} - a \div \left\{ \text{half} \left(n - 1 \right) \right\}$$

124) Example

On an expedition to seize his enemy's elephants, a king marched two yojanas the first day. Say, intelligent calculator, with what increasing rate of daily march did he proceed, since he reached his foe's city, a distance of eighty yojanas, in a week?

Statement: first 2, sum 80, period 7, increase ?

common difference ? **Divide** *80 by 7*

Subtract *2;* *'66 by 7'*

Divide *by half* of *6;* *'22 by 7'* $\frac{\mathbf{22}}{\mathbf{7}}$

Common difference, $\frac{\mathbf{22}}{\mathbf{7}}$.

125) ... Period

From the sum of the progression multiplied by twice the common increase, and added to the square of the difference between the first term and half that increase, the square root being extracted, this root less the first term and added to the [above-mentioned] portion of the increase, being divided by the increase, is pronounced to be the period.

period ?

Multiply *sum* by *'twice increase'*	S x $2d$
Add *sq* of *'diff* of *first* & *half increase'*	$+\left\lvert a-\left(\frac{1}{2}d\right)\right\rvert^2$
Subtract *first term* from its *root;* add *half increase*	$\sqrt{\ }$, $-a+\frac{1}{2}d$
Divide *by increase*	$\div d$

126) Example

A person gave three drammas on the first day, and continued to distribute alms increasing by two [a day]; and he thus bestowed on the priests three hundred and sixty drammas: say quickly in how many days?

Statement: first 3; increase 2; period?; sum 360

Multiply	*360* by *'twice 2;*	*1440*
Add	*$(3-1)^2$;*	*1444*
Subtract	'rt 1444', *(3 + 1)/2;*	*36*
Divide	*by 2*	*18*

Multiply *sum* by '*twice increase*',	360 x 4,	1440
Difference of '*half increase*' from '*first*',	2	
Add sq of this diff to the above product,	1444	
Sq rt this,	38	
From this take away the above diff,	38 – 2,	36
Divide by increase to get period,	18	
In 18 days.		

SECTION II

GEOMETRICAL PROGRESSION

A PROGRESSION, THE INCREASE BEING A MULTIPLIER

127) Sum

The period being an uneven number, subtract one, and note "multiplicator"; being an even one, halve it, and note "square:" until the period be exhausted. Then the produce arising from multiplication and squaring [of the common multiplier] in the inverse order from the last, being lessened by one, the remainder divided by the common multiplier less one, and multiplied by the intial quantity, will be the sum of a progression increasing by a common multiplier.

period, *n;* common multiplier, *r* ; first (initial quantity), *a*; sum, *S*.

sum?

Find *(multiplier)*$^{(period)}$	r^n
Lessen *by 1*	-1
Divide by 'multiplier, less 1'	$\div(r-1)$
Multiply *by* initial quantity.	$x\ a$